Algebra Terms and Rules

Algebra uses letters $(x, y, etc.)$ to represent unknown quantities (variables) that we need to solve for. Algebra is used to solve problems in everyday life as well as being used in most modern-day occupations. Mastery of its basic terms and rules will make learning this important subject much easier.

A natural number (also called a counting number or a whole number) is a member of the set of positive integers. Examples: 1, 2, 3, 4, 5, ...

An integer is a whole number that can be written without a fractional part.
Examples: ..., -3, -2, -1, 0, 1, 2, 3, ... (These numbers are NOT integers: 2.3, $5\frac{1}{2}$, $\sqrt{2}$.)

A rational number is formed by dividing one integer by another integer (denominator $\neq 0$). It is the ratio of two integers, or the fraction formed by dividing two integers.
Examples: $\frac{2}{5}$, $\sqrt{16} = 4 = \frac{4}{1}$, $0.91 = \frac{91}{100}$, $\frac{1}{3} = 0.33333... = 0.\overline{3}$. (The bar over the $\overline{3}$ indicates repetition. Repeating numbers are rational.)

Zero is a rational number without a sign. Any number multiplied by zero = 0. Zero divided by any number (except zero) = 0. Division of any number by zero is undefined and is not allowed. In computer programs, division by zero will usually cause an error.

An irrational number cannot be formed as a fraction of integers. Unlike rational numbers which either end or repeat, irrational numbers never end and do not repeat.
Examples: $\sqrt{2} = 1.414213562...$, $\sqrt{3} = 1.732050808...$, $e = 2.718281828...$, $\pi = 3.141592653...$

Real numbers comprise the sets of both rational numbers and irrational numbers.

Complex numbers (also called $phasors$) include the symbol i which stands for the word 'imaginary' and where $i = \sqrt{-1}$. Complex numbers are used in engineering and in the sciences. In electrical engineering, complex numbers are used to calculate the phase and magnitude of oscillating voltages or currents in circuits. Specialized computer programs called $circuit$ $simulators$ are widely used by engineers (and hobbyists) to analyze electronic circuits automatically. (LTspice IV is a free circuit simulator that you can download and experiment with.)
Example: $x + iy$, where x and y are real numbers and i is called the imaginary part.

© 2016 Robert D. Schoening

A term is that part of an expression that is separated by + or - signs. A term consists of the products and/or quotients of numbers and letters representing numbers.
Examples: $3x^2$, $53xy$, $a^n b^m$, $\dfrac{8x}{5y^2}$. This expression $4x^2 + 3x + 5$ has 3 terms.

Like terms differ only by numerical coefficient; the exponent of each variable is the same. Like terms can be added and subtracted; unlike terms cannot.
Examples of like terms: $5x^3y^2z^2$ and $14x^3y^2z^2$. (Only the coefficients 5 and 14 differ.)
Examples of unlike terms: $5x^3y^2z^2$ and $5x^3y^3z^2$. (The exponent of y differs.)

Exponents are a special notation that indicate repeated multiplication of a number by itself a certain number of times (the exponent number of times). For example, x^5 means x with an exponent of 5. It means to multiply x by itself 5 times: $x^5 = x \cdot \cdot x \cdot x \cdot x \cdot x$. So if we let $x = 2$, then $x^5 = 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. Other ways of saying x with an exponent of 5 are "x raised to the power of 5" or "x to the $5th$".

A polynomial is an expression consisting of variables and their coefficients and the operations of addition, subtraction, multiplication, and non-negative integer exponents. A polynomial can have one term or many terms. Each term must be integral (an integer) and rational.
Example: $3x^2 + 5x + 10$.

A monomial is a polynomial with one term.
Examples: $3x^2$, $5x^3y^2z^2$

A binomial is a polynomial with two terms, each of which is a monomial.
Example: $3x^2 + 5x$

A trinomial is a polynomial with three terms, each of which is a monomial.
Example: $3x^2 + 5x + 14$.

A multinomial is a polynomial with more than one term

The degree of a monomial is the sum of all the variable exponents in the term. Therefore, the degree of $5x^3y^2z^2$ is $3 + 2 + 2 = 7$. But the degree of a polynomial is the same as the degree of the $term$ with the highest degree. For example, the degree of $6x^3y^2z + 4x^5y^3 + 47$ is 8 because the term with the highest degree $(4x^5y^3)$ has degree 8 $(5 + 3 = 8)$. The degree of a constant such as 47 is zero.

A constant polynomial has degree 0. A linear polynomial has degree 1. A quadradic polynomial (or simply a quadratic) has degree 2. A cubic polynomial has degree 3.

© 2016 Robert D. Schoening www.worksheetsandwalkthroughs.com/codelagebra.php

① Commutative property for addition

$a + b = b + a, \qquad 2 + 3 = 3 + 2 = 5$

② Associative property for addition

$a + b + c = a + (b + c) = (a + b) + c, \qquad 2 + 3 + 4 = 2 + (3 + 4) = (2 + 3) + 4 = 9$

③ Commutative property for multiplication

$a \cdot b = b \cdot a, \qquad 2 \cdot 3 = 3 \cdot 2 = 6$

④ Associative property for multiplication

$abc = a(bc) = (ab)c, \qquad 2 \cdot 3 \cdot 4 = 2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4 = 24$

⑤ Distributive property for multiplication over addition

$a(b + c) = ab + ac, \qquad 2 \cdot (3 \cdot 4) = 2 \cdot 3 + 2 \cdot 4 = 14$

⑥ Addition Property of Equality

If $a = b$, then $a + c = b + c$.
You can add the same number (c) to both sides of an equation and get the same answer.

⑦ Subtraction Property of Equality

If $a = b$, then $a - c = b - c$.
You can subtract the same number (c) from both sides of an equation and get the same answer.

⑧ Multiplication Property of Equality

$a = b$, then $a \cdot c = b \cdot c$.
You can multiply both sides of an equation by the same number (c) and get the same answer.
This is especially useful for eliminating the fractional part of an equation.

Ex 1. $\frac{x}{3} = 12$. Multiplying both sides by 3: $\quad \cancel{3} \cdot \frac{x}{\cancel{3}} = 3 \cdot 12$, so $x = 36$.

Ex 2. $\frac{2x}{3} = 12$. Multiplying both sides by $\frac{3}{2}$: $\quad \frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} x = \frac{6}{6} x = 1 \cdot x = \frac{3}{2} \cdot 12$, so $x = 18$.

© 2016 Robert D. Schoening www.worksheetsandwalkthroughs.com/codelagebra.php

(9) Laws of Exponents

$a^1 = a, \qquad a^5 = a \cdot a \cdot a \cdot a \cdot a, \qquad a^m a^n = a^{m+n}, \qquad (ab)^m = a^m b^m, \qquad (a^m)^n = a^{mn},$

$a^{m/n} = \sqrt[n]{a^m}, \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \qquad a^{-m} = \frac{1}{a^m}, \qquad \frac{1}{a^{-m}} = a^m,$

$\frac{x^a}{x^b} = x^{a-b} = \frac{1}{x^{b-a}}, \qquad 1^n = 1.$

$If \;\; a \neq 0, \qquad \frac{a^m}{a^n} = a^{m-n}, \qquad a^0 = 1.$

$(-4)^2 = (-4)(-4) = 16, \;\; but - 4^2 = -1 \cdot 4 \cdot 4 = -16.$

$5 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y = 5x^4y^2 \qquad (x-y)(x-y)(x-y)(x-y) = (x-y)^4$

(10) Zero

(Division by zero is undefined and not allowed.)

If $a \neq 0: \qquad \frac{0}{a} = 0, \qquad a^0 = 1, \qquad a^{-m} = \frac{1}{a^m}, \qquad 0^a = 0, \qquad (0^0 = indeterminate).$

(11) Fractions

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}, \qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \qquad \frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c}, \qquad \frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b},$

$$\frac{(a/b)+(c/d)}{(e/f)+(g/h)} = \frac{(a/b)+(c/d)}{(e/f)+(g/h)} \cdot \frac{bdfh}{bdfh} = \frac{(ad+bc)fh}{(eh+fg)bd}$$

© 2016 Robert D. Schoening www.worksheetsandwalkthroughs.com/codelagebra.php

(12) The Binomial Theorem (Binomial Expansion)

For any positive integer n (1,2,3,...),

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1\cdot 2} + a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}a^{n-3}b^3 + ... + nab^{n-1} + b^n.$$

Examples:

$(a+b)^1 = a+b,$
$(a+b)^2 = a^2 + 2ab + b^2,$
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + 3b^3,$
$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$
$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5,$
$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6,$
$(a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7,$
$(a+b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8,$
$(a+b)^9 = a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + 126a^5b^4 + 126a^4b^5 + 84a^3b^6 + 36a^2b^7 + 9ab^8 + b^9$

© 2016 Robert D. Schoening www.worksheetsandwalkthroughs.com/codelagebra.php